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Optimum performances of longitudinal convective fins with symmetrical and asymmetrical profiles

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Abstract

In the present work, the heat transfer performance of optimized dissipators with longitudinal fins of asymmetrical cross section is investigated and compared with that of optimized dissipators with symmetrical fins. In particular, the problem of optimizing the shape and the spacing of the fins of a thermal dissipator cooled by a fluid in laminar flow is studied by assigning two different polynomial lateral profiles to the fins. A finite element model is proposed to determine velocity and temperature distributions and is employed in a genetic algorithm to find the dissipator geometries which make the heat transfer coefficient as high as possible under different conditions. Some examples of optimized geometries are finally shown and discussed. © 1999 Elsevier Science Inc. All rights reserved.

Notation

a	fin height (m)
b	fin basement thickness (m)
c_p	coolant specific heat (J/kg K)
d	distance between the fin base and the opposite flat wall (m)
e	width of the portion of conduit section (m)
E_c	compared effectiveness
f_1, f_2	profile functions (m)
h	global heat transfer coefficient (W/m^2 K)
h_r	global heat transfer coefficient of a reference flat wall conduit (W/m^2 K)
k_c	thermal conductivity of the coolant (W/m K)
M_A	surface averaging matrix (m^2)
M_M	momentum transfer matrix
M_H	heat transfer matrix
n	polynomial order
Nu_e	equivalent Nusselt number
p	generalized pressure (N/m^2)
q''	heat flux per unit of surface uniformly imposed on the flat side of the finned plate (W/m^2)
T_b	bulk temperature of the coolant (K)
T_c	temperature of the coolant (K)
T_f	temperature of the finned plate (K)
T_{max}	maximum temperature of the finned plate (K)
u	coolant velocity (m/s)
w_t	total coolant volume flow rate (m^3/s)
x	longitudinal coordinate (m)
y	coordinate parallel to fin height (m)
z	coordinate orthogonal to fin height (m)

Greek

α	normalized height of the fins, a/d
β	normalized thickness of the finned plate base, b/d
γ	ratio of finned plate to coolant thermal conductivity
ϵ	normalized width of the portion of conduit section, e/d
ζ	normalized hydraulic resistance, defined by Eq. (25)
η	normalized coordinate parallel to fin height, y/d
μ	dynamic viscosity (Pa s)
ρ	coolant density (kg/m^3)
$\overline{\sigma}$	finned plate normalized average thickness, defined by Eq. (30)
ϕ_1, ϕ_2	normalized profile functions; f_1/d, f_2/d
ϕ_{1i}, ϕ_{2i}	fin profile describing parameters

1. Introduction

In many engineering applications, finned dissipators are commonly used to promote high heat fluxes from small components having a limited heat transfer surface. During the last few years, the need to reduce the volume and the weight of thermal dissipators has become even more important. For new applications, such as in the electronic industry (Bar-Cohen and Kraus, 1990) or in the compact heat exchanger field (Kays and London, 1984), even smaller and lighter dissipators have been, in fact, required. Therefore, the problem of optimizing the geometry of finned dissipators in order to increase the heat transfer effectiveness and reduce the dimensions and the weight has been studied by many researchers.

To maximize the heat flux removed through finned surfaces, a variety of fin profiles has been studied since the 1920s

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(Schmidt, 1926; Duffin, 1959; Maday, 1974; Tsukamoto and Seguchi, 1984; Snider and Kraus, 1987; Chung and Iyer, 1993). Parabolic, triangular, undulated optimized profiles have been proposed. For longitudinal fins, under particular conditions, some of them have been demonstrated to have a significantly improved effectiveness (Snider et al., 1990; Spiga and Fabbri, 1994; Fabbri and Lorenzini, 1995; Fabbri, 1997). Nevertheless, for many situations, an ultimate solution has not yet been found to the problem of optimizing the profiles of the fins.

For finned dissipators cooled by forced convection, the heat transfer effectiveness depends on different factors, which often are interdependent. The most important are the local convective heat transfer coefficient, the extension of the heat transfer surface between the solid and the coolant fluid and the fin conductance. The local convective heat transfer coefficient depends on the velocity distribution of the coolant fluid induced by the fin spacing and shape, which also affect the heat transfer surface and the fin conductance. Moreover, to increase the heat transfer surface, the extension of the fin must be augmented. Therefore, if the fin weight is constrained, the fin thickness must be reduced. Nevertheless, for a given value of the thermal conductivity of the fin material, it is necessary to reduce the height and to increase the thickness of the fins in order to enhance the conductance.

The problem of optimizing the geometry of a dissipator with longitudinal fins having a symmetrical cross section under laminar convection conditions has been studied in a previous work (Fabbri, 1998). In this case, we demonstrated that the local heat transfer coefficient on the finned surface is very sensitive to the fluid dynamic conditions determined by the fin profile. As a result, by optimizing the fin profile, the improvements obtained in the heat transfer effectiveness of the dissipators depend much more on the increase in the local heat transfer coefficient than on the fin conductance enhancement or surface extension.

Longitudinal fins having symmetrical lateral profiles have been investigated, as in our previous work, in most of the studies performed on the optimization of the fin shape. The symmetry of the lateral profiles simplifies the treatment of the problem. Boundary conditions, in fact, can be more easily imposed. Moreover, a less extended domain can be studied, that is particularly convenient if the thermal state of the system is determined in a numerical way. Nevertheless, in many practical applications, the adoption of symmetrical profiles does not provide the best solution in terms of heat transfer effectiveness. This is particularly true when binding constraints are imposed on the dissipator structural integrity and weight. Therefore, in the present work we investigate the improvements which can be obtained under laminar flow conditions in the heat transfer effectiveness of optimized finned dissipators by assigning asymmetrical lateral profiles to the fins.

2. The mathematical model

Let us consider a coolant fluid which passes in laminar flow through a conduit delimited by a flat insulated surface and a finned plate as in Fig. 1(a). All fins are identical and have an asymmetrical cross section. Moreover, a heat flux q'' is uniformly imposed on the flat side of the finned plate.

Let us choose an orthogonal coordinate system, where the x axis is directed in the coolant flow direction and the y axis is orthogonal to the flat conduit wall. Moreover, let Ω_1 and Ω_2 be the lines which pass through the middle points of two adjacent fins, e the distance in z direction between Ω_1 and Ω_2, a the fin height in the y direction, b the base thickness, d the distance between the base and the flat insulated surface, and $f_1(y)$ and $f_2(y)$ arbitrary functions which describe the two lateral fin profiles.

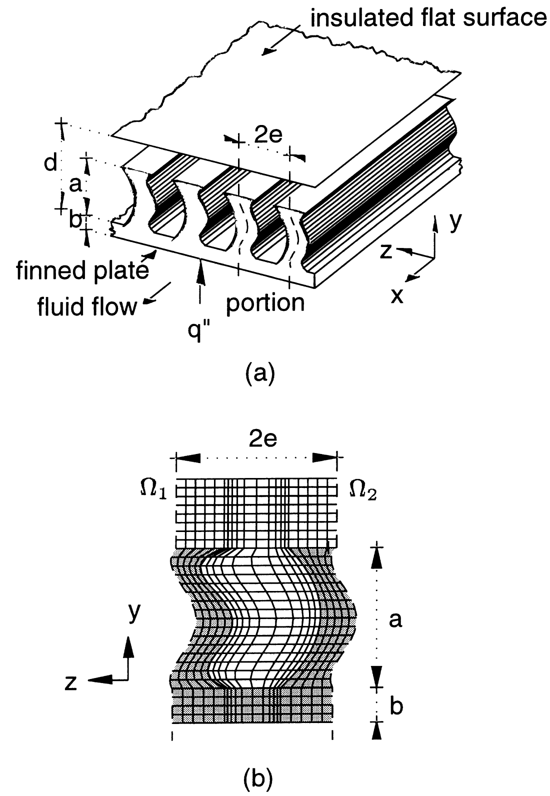

Fig. 1. Geometry of the heat removing system: (a) view of the finned conduit, (b) portion of the transversal section subdivided in finite elements.

Since the dynamic and thermal state of the system is periodic in the z direction, the heat transfer performance can be investigated by limiting the analysis to the portion of the conduit cross section delimited by lines Ω_1 and Ω_2. The same idealizations invoked by Fabbri (1998) are assumed:

– the system is in steady state;
– velocity and temperature profiles are completely developed;
– fluid and solid properties are uniform;
– viscous dissipation within the fluid is negligible;
– natural convection is negligible in regard to the forced convection.

Under these conditions the coolant flow is described by the following equation:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}, \tag{1}$$

p being the generalized pressure, which includes the gravitation potential, and μ the dynamic viscosity. For the case where the fins are asymmetrical, Eq. (1) must be integrated by imposing the following boundary conditions: the velocity is zero on the contact surface between the fluid and the finned plate and

$$u[y, \omega_1(y)] = u[y, \omega_2(y)] \quad \forall\, a \leqslant y \leqslant d, \tag{2}$$

$$\left[\frac{\partial u}{\partial N}\right]_{[y,\omega_1(y)]} = \left[\frac{\partial u}{\partial N}\right]_{[y,\omega_2(y)]} \quad \forall\, a \leqslant y \leqslant d, \tag{3}$$

where functions $\omega_1(y)$ and $\omega_2(y)$ provide the value of the angular coordinate in Ω_1 and Ω_2, respectively, and N is the coordinate which is normal to the two lines.

In the coolant, the temperature must satisfy the following energy balance equation:

$$\frac{\partial^2 T_c}{\partial y^2} + \frac{\partial^2 T_c}{\partial z^2} = \frac{\rho c_p}{k_c} u \frac{\partial T_c}{\partial x}, \tag{4}$$

ρ, c_p and k_c being the coolant density, specific heat and thermal conductivity, respectively. In the finned plate, the temperature must instead satisfy the Laplace equation:

$$\frac{\partial^2 T_f}{\partial y^2} + \frac{\partial^2 T_f}{\partial z^2} = 0. \tag{5}$$

For the case where fins are asymmetrical, Eqs. (4) and (5) must be integrated by imposing the following boundary conditions: the temperature and the heat flux in normal direction are identical in the solid and in the fluid on the contact surface, the heat flux in the normal direction is zero on the insulated flat surface and is equal to q'' on the flat side of the finned plate, and

$$T_c[y, \omega_1(y)] = T_c[y, \omega_2(y)] \quad \forall\ a \leqslant y \leqslant d, \tag{6}$$

$$T_f[y, \omega_1(y)] = T_f[y, \omega_2(y)] \quad \forall\ -b \leqslant y \leqslant a, \tag{7}$$

$$\left[\frac{\partial T_c}{\partial N}\right]_{[y,\omega_1(y)]} = \left[\frac{\partial T_c}{\partial N}\right]_{[y,\omega_2(y)]} \quad \forall\ a \leqslant y \leqslant d, \tag{8}$$

$$\left[\frac{\partial T_f}{\partial N}\right]_{[y,\omega_1(y)]} = \left[\frac{\partial T_f}{\partial N}\right]_{[y,\omega_2(y)]} \quad \forall\ -b \leqslant y \leqslant a. \tag{9}$$

It also is necessary to impose a temperature value in one point of the studied domain.

Similarly to the case of symmetrical fins (Fabbri, 1998), due to the complexity of the problem it is convenient to numerically determine velocity and temperature distributions resorting, for example, to a finite element method. The same shape of that case can be assigned to the elements (see Fig. 1(b)) and the same interpolation can be used to approximate the velocity and the temperature in each element.

Let us suppose, in the first instance, that condition (2) is not necessarily verified and the momentum flux through contours Ω_1 and Ω_2 is zero. In this way, the finite element fluid dynamic problem becomes similar to that studied in Fabbri (1998). Therefore, from a balance of viscous and pressure forces acting on each node, the following system of equations is obtained:

$$M_M \cdot U = \frac{1}{\mu}\frac{dp}{dx} A, \tag{10}$$

U and A being vectors containing the velocity of each node and the portion of the cross section associated to each node, respectively, and M_M a momentum transfer matrix.

At this stage, to impose the periodicity of the velocity, the system of equations (10) can be reduced by attributing the contributions of the nodes on a contour line, for example Ω_2, to those on the other. In particular, to verify conditions (2) and (3), the matrix M_M and the vectors U and A must be modified in the following way:

$$\hat{U} = \begin{pmatrix} U_1 \\ U_3 \end{pmatrix}, \tag{11}$$

$$\hat{A} = \begin{pmatrix} A_1 + A_2 \\ A_3 \end{pmatrix}, \tag{12}$$

$$\hat{M}_M = \begin{pmatrix} M_{M11} + M_{M12} + M_{M21} & M_{M13} + M_{M23} \\ M_{M31} + M_{M32} & M_{M33} \end{pmatrix}, \tag{13}$$

index 1 and 2 referring to the nodes on the lines Ω_1 and Ω_2, and index 3 to all other nodes. Moreover, the vectors $\hat{U}_1$ $\hat{A}_1$ and the matrix $\hat{M}_M$ can be partitioned by distinguishing nodes where velocity is known:

$$\hat{M}_{Muu} \cdot \hat{U}_u = \frac{1}{\mu}\frac{dp}{dx}\hat{A}_u, \tag{14}$$

index u referring to the nodes where the velocity is unknown. Since the known velocities are zero, their contribution has not been taken into account. For the case of asymmetrical fins, the velocity distribution in the cross section is determined by solving the system of equations (14).

Supposing that conditions (6) and (7) are not necessarily verified and the heat flux through contours Ω_1 and Ω_2 is zero, the finite element heat transfer problem also becomes similar to that studied in Fabbri (1998). From an overall energy balance, the following system of equations is then obtained:

$$M_H \cdot T = N, \tag{15}$$

$$N = \frac{q''}{k_c}\left(\frac{2e}{w_t} M_A \cdot U - L\right), \tag{16}$$

M_H and M_A being heat transfer and surface integration matrices, respectively, w_t the total volume flow rate through the portion of the finned tube section, and L a vector containing the perimeter crossed by q'' associated to each node. Vector U now includes the velocities of the nodes of the solid, which are zero.

To verify conditions (6)–(9), the matrix M_H and the vectors T and N must now be modified in the following way:

$$\hat{T} = \begin{pmatrix} T_1 \\ T_3 \end{pmatrix}, \tag{17}$$

$$\hat{N} = \begin{pmatrix} N_1 + N_2 \\ N_3 \end{pmatrix}, \tag{18}$$

$$\hat{M}_H = \begin{pmatrix} M_{H11} + M_{H12} + M_{H21} & M_{H13} + M_{H23} \\ M_{H31} + M_{H32} & M_{H33}, \end{pmatrix}, \tag{19}$$

index 1, 2 and 3 referring to the same nodes as in Eqs. (13)–(15). By assigning an arbitrary temperature to a node, distinguishing it from the others, whose temperature is unknown, and consequently partitioning the matrix $\hat{M}_H$ and the vectors $\hat{T}$ and $\hat{N}$, the following system is derived:

$$\hat{M}_{Huu} \cdot \hat{T}_u = \hat{N}_u - \hat{M}_{Hun} \cdot \hat{T}_n, \tag{20}$$

index n and u referring here to the node to which the arbitrary temperature has been assigned and to the others, respectively. After solution of the system of equations (20), the temperature distribution in the cross section of the conduit with asymmetrical fins is obtained as a function of T_n.

Bulk temperature, global heat transfer, equivalent Nusselt number, compared effectiveness and normalized hydraulic resistance can finally be defined as in Fabbri (1998) and calculated as follows:

$$T_b = \frac{1}{w_t}\sum_i \sum_k m_{Aik} u_k t_k, \tag{21}$$

$$h = \frac{q''}{T_{max} - T_b}, \tag{22}$$

$$Nu_e = \frac{h\,2d}{k_c}, \tag{23}$$

$$E_c = \frac{q''}{q''_r} = \frac{h}{2.692 k_c \sqrt[3]{\zeta}/d}, \tag{24}$$

$$\zeta = \frac{(-dp/dx)}{(w_t/2e)} \Bigg/ \frac{12\,\mu}{d^3} \tag{25}$$

where m_{Aik}, u_k, and t_k are the elements of M_A, U, and T, respectively, the summation index i is extended to all nodes of the coolant, T_{max} is the maximum temperature obviously occurring on the surface where q'' is imposed, and q''_r is the heat flux dissipated through a side of a reference flat wall conduit (Shah and London, 1974) with the same hydraulic resistance as the finned conduit. Due to the linearity of the system, coefficient h does not depend on the value arbitrarily assigned to T_n.

3. Geometry optimization

In this work the geometry of the system described in Section 2 will be optimized under different conditions, in order to maximize the equivalent Nusselt number or the compared effectiveness. The highest heat flux is dissipated for given distance d between the fin basement and the insulated flat surface in the first case, and for given hydraulic resistance in the second one. The condition of constrained finned plate volume will also be taken into consideration.

Parameters a, b, d, e and the profile functions $f_1(y)$ and $f_2(y)$ describe the geometry of the finned conduit. In the studied domain, z coordinate is equal to $f_1(y)$ on a lateral fin profile and to $2e - f_2(y)$ on the other. Dimensionless variables can be obtained by normalizing all geometrical parameters by d:

$$\alpha = \frac{a}{d}, \quad \beta = \frac{b}{d}, \quad \epsilon = \frac{e}{d}, \quad \phi_1(\eta) = \frac{f_1(\eta d)}{d},$$
$$\phi_2(\eta) = \frac{f_2(\eta d)}{d}, \quad \eta = \frac{y}{d}. \tag{26}$$

To the profile functions $\phi_1(\eta)$ and $\phi_2(\eta)$ a polynomial form as in Fabbri (1998) can be assigned. Moreover, the values assumed by the two functions in $n+1$ points can be chosen as the fin profile describing parameters, n being the polynomial order:

$$\phi_{1i} = \phi_1\left(\frac{i}{n}\alpha\right), \quad \phi_{2i} = \phi_2\left(\frac{i}{n}\alpha\right) \quad \forall\, i = 0, 1, \ldots, n. \tag{27}$$

The dynamic and thermal behavior of the finned conduit does not obviously depend on the origin of the coordinate system. Therefore, this latter can be translated in the z direction in order to let ϕ_{10} always be equal to ϕ_{20}. This allows the number of the describing parameters to be reduced by one unit. Moreover, the same thermal performance is presented by two finned tubes whose cross section is specular. By imposing, for example, that the derivative of ϕ_1 is not negative, the number of possible finned tube geometries can then be halved.

To find the combinations of parameters α, β, ϵ and ϕ_i which allow the best heat transfer performance to be obtained even in case the dissipator volume is constrained, a genetic algorithm (Queipo et al., 1994; Fabbri, 1997) similar to that utilized in Fabbri (1998) can be successfully employed. The following variations must be applied.

Let $\phi_3(\eta)$ be the fin thickness given by the sum of $\phi_1(\eta)$ and $\phi_2(\eta)$. This parameter must be no less than a lower limit value θ_{min} to ensure the structural integrity of the fin, and no more than an upper limit value θ_{max} to allow a uniform distribution of the coolant in the space between the fins. If, after parameter reproduction, the fin thickness is found to exceed these limits, parameters ϕ_{1i} must be changed in the following way

$$\phi_{3i} = \begin{cases} \phi_{3\,max} - \frac{\phi_{3\,max} - \theta_{min}}{\phi_{3\,max} - \phi_{3\,min}}[\phi_{3\,max} - \phi_3(\alpha\, i/n)] & \text{if } \phi_{3\,min} < \theta_{min} \\ \phi_{3\,min} + \frac{\theta_{max} - \phi_{3\,min}}{\phi_{3\,max} - \phi_{3\,min}}[\phi_3(\alpha\, i/n) - \phi_{3\,min}] & \text{if } \phi_{3\,max} > \theta_{max} \end{cases}$$
$$\forall\, i = 0, 1, \ldots, n, \tag{28}$$

$$\hat{\phi}_{1i} = \phi_{1i} + \phi_{3i} - \phi_3(\alpha\, i/n), \tag{29}$$

where $\phi_{3\,min}$ and $\phi_{3\,max}$ are the minimum and the maximum values which $\phi_3(\eta)$ was found to assume for η varying between 0 and α, and $\hat{\phi}_{1i}$ being the new parameter values.

4. Results

Some geometry optimizations have been carried out in order to find the finned plate geometries which maximize the equivalent Nusselt number or the compared effectiveness. Performances of asymmetrical optimized fins have been compared with those of symmetrical ones found with the procedure described in Fabbri (1998).

To determine velocity and temperature distributions in finned conduits with asymmetrical fins, a grid of 32×52 elements (33×53 knots) was employed in the finite element model. For the model predictions, the same grid dependence as in Fabbri (1998) has been observed for the same domains. In particular, a grid of 40×52 elements produced in Nu_e and E_c alterations of less than 0.1% and 0.2%, respectively and a grid of 32×65 elements resulted in alterations of less than 0.05% and 0.1%, respectively. In the genetic algorithm, α has been constrained to 0.75 and the maximum displacement in z direction between the middle points of the fin has been imposed to be no greater than 2ϵ.

Optimizations of the geometries of conduits with asymmetrical fins have been carried out for the same cases investigated in Fabbri (1998) (for $\alpha = 0.75$). In every case, symmetrical fins have been found as performing the best, both in terms of Nu_e and E_c. On the contrary, better performances have been presented by asymmetrical fins in some cases where ϵ was constrained. In particular, it has been observed that, under different conditions, if the fins are constrained to be spaced far apart compared to those of the optimum geometry obtained by imposing no constraint on ϵ, then optimum profiles resulted as being asymmetrical.

This result is particularly interesting since, in many practical applications, the fin spacing must be augmented for different reasons. Dissipating plates with symmetrical or asymmetrical fins can be produced by extrusion of melted metal through appropriate dies or by utilizing composite moulds. The cost of producing dies or moulds, which must be frequently substituted, depends on the number of fins and the complexity of the profiles. Therefore, fins spaced far a part are less expensive to be produced. Moreover, if the dissipator volume is constrained, sparse fin can be thicker and more resistant.

In Fig. 2 optimized geometries with symmetrical (a and c) and asymmetrical fins obtained by maximizing Nu_e (a and b) and E_c (c and d) are shown for n ranging from 0 to 4, ϵ constrained to 0.5 α, and γ equal to 300. The value chosen for ϵ is higher than those of the optimum geometries found in Fabbri (1998). It is evident that, under the considered conditions, higher order polynomial profiles still perform better. Moreover, performances of the asymmetrical fins are significantly better both in terms of equivalent Nusselt number and compared effectiveness, mainly when n is high. Lastly, asymmetrical fins require a smaller solid volume, as indicated by the average normalized thickness of the finned plate $\overline{\sigma}$:

$$\overline{\sigma} = \beta + \frac{1}{2\epsilon}\int_0^\alpha (\phi_1 + \phi_2)\,\mathrm{d}\eta. \tag{30}$$

It must be noticed that the equivalent Nusselt numbers of first and second order asymmetrical fins are very close to those of third and fourth order symmetrical fins, respectively. Therefore, in terms of Nu_e, optimum performances comparable with those of symmetrical fins can be obtained with simpler

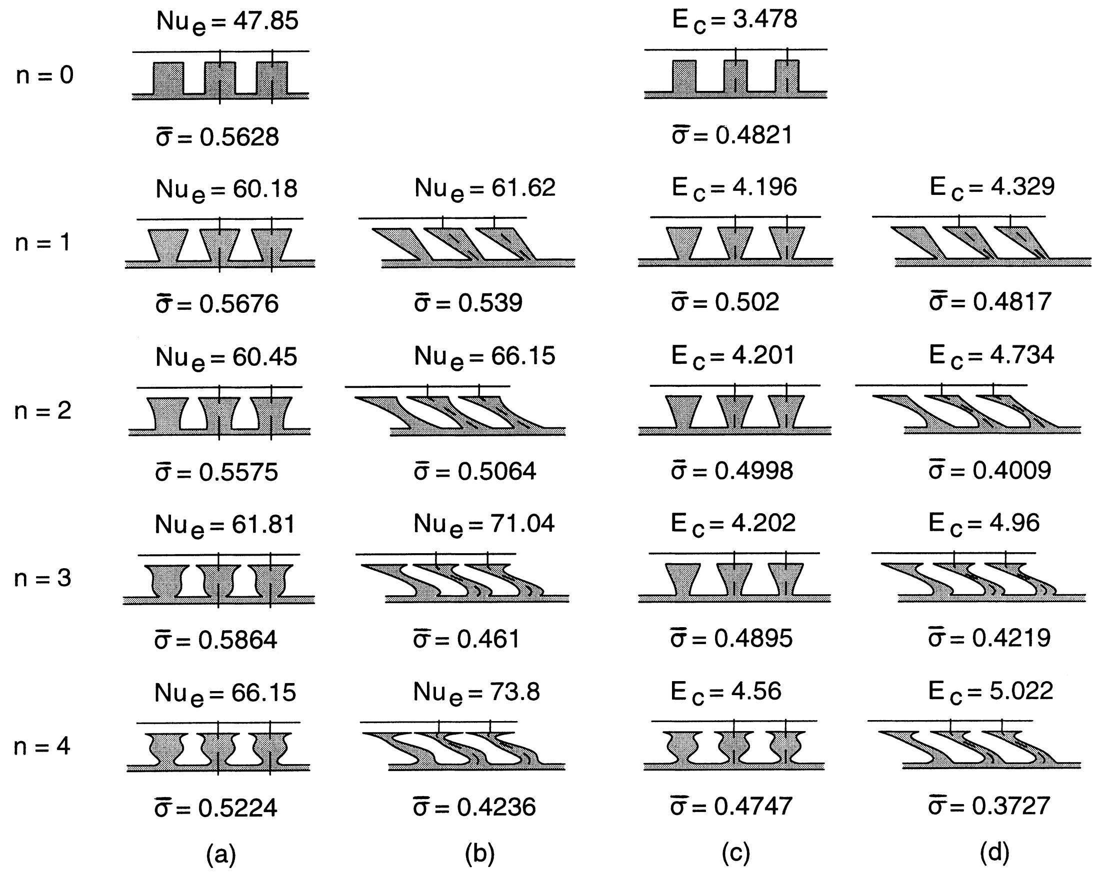

Fig. 2. Finned conduit geometries with symmetrical (a and c) and asymmetrical fins obtained by maximizing Nu_e (a and b) and E_c (c and d) for ϵ constrained to 0.5α and γ equal to 300.

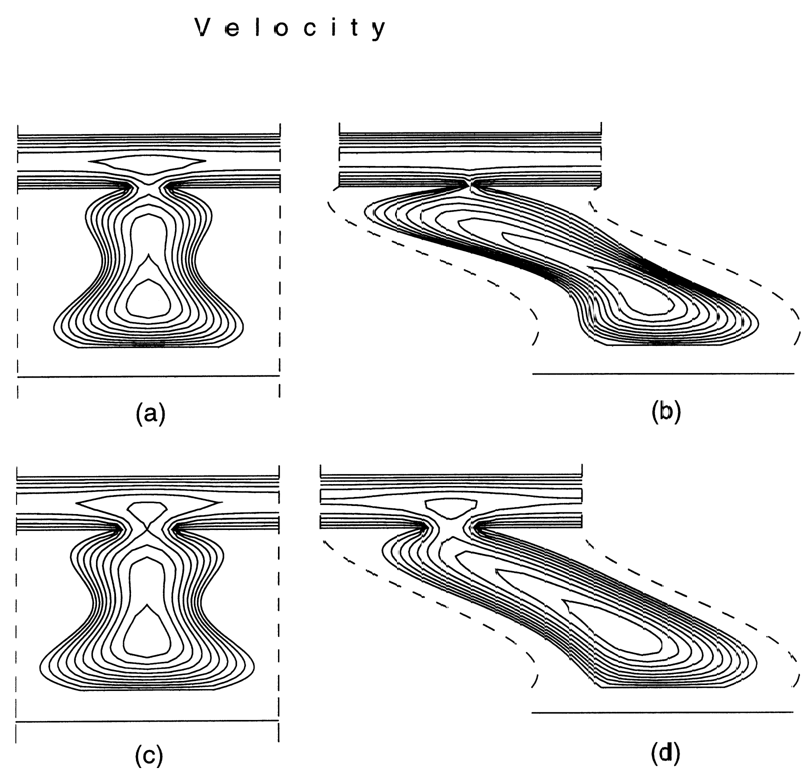

Fig. 3. Velocity distributions in the transversal section of the geometries of Fig. 2 with n equal to 4. Curves are drawn every 10% of the maximum velocity.

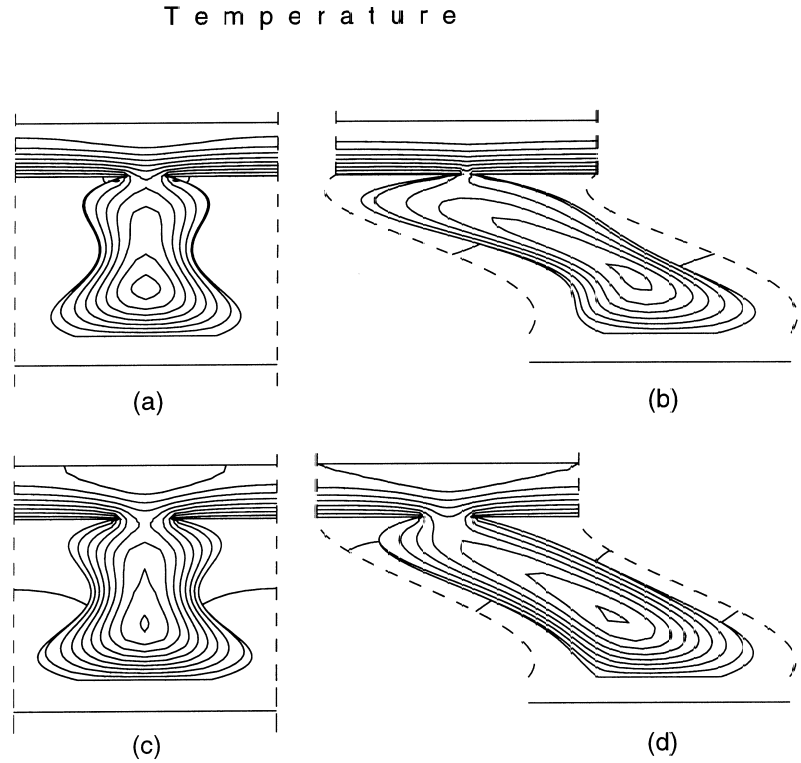

Fig. 4. Temperature distributions in the transversal section of the geometries of Fig. 2 with n equal to 4. Curves are drawn every 10% of the difference between the maximum and minimum temperature.

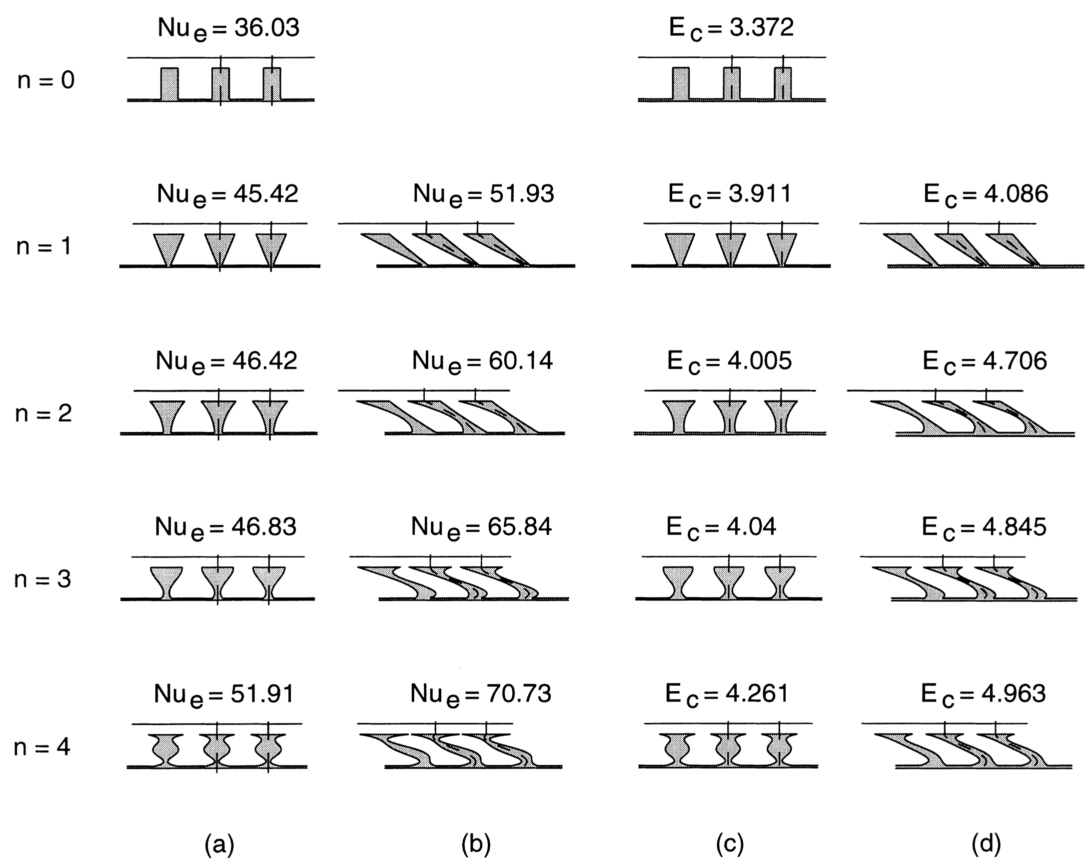

Fig. 5. Finned conduit geometries with symmetrical (a and c) and asymmetrical fins obtained by maximizing Nu_e (a and b) and E_c (c and d) for γ equal to 300, ϵ constrained to 0.5α, $\bar{\sigma}$ to 0.3, and β to be more than 0.05.

asymmetrical profiles. Moreover, in terms of E_c, performances of first and second order asymmetrical fins are considerably better compared with those of third and fourth order symmetrical fins, respectively. This is particularly interesting, since the complexity of the profiles affects the production costs as observed above.

Velocity and temperature distributions in the optimized conduit of Fig. 2 with fourth order profiles fins are shown in Figs. 3 and 4, respectively. Temperature gradients near the surface of symmetrical and asymmetrical fins are comparable. Therefore, the better performances of asymmetrical fins are mainly due to the larger extension of the heat transfer surface between solid and fluid and are not so much affected by the violation of those idealizations, introduced in the mathematical model, which in practical applications results in lowering the thermal gradients (Fabbri, 1998; Huang and Shah, 1992).

Since optimum asymmetrical fins require a smaller solid volume it is interesting to compare their performances with those of symmetrical fins for a given amount of material available for the finned plate. It has been demonstrated (Fabbri, 1998) that, when the solid volume is constrained, the fin basement can become very thin in the optimized finned plates. On the contrary, in most practical applications, the fin basement can not be too thin, in order to ensure the structural integrity of the dissipator. Therefore, it also is interesting to constrain the basement thickness β to be no less than a limit value.

In Fig. 5, optimized geometries obtained for $\overline{\sigma}$ constrained to 0.3 and β constrained to be no less than 0.05 are shown. By reducing the available solid volume, the Nusselt number and the compared effectiveness decrease considerably less for the asymmetrical fin optimum geometries than for the symmetrical one. As a result, the maximum equivalent Nusselt number obtainable with fourth order asymmetrical profiles is 36.26% greater than with fourth order symmetrical profiles and 96.31% greater than with zero order profiles. Moreover, in the compared effectiveness, fourth order asymmetrical profiles provide a 16.48% increment in comparison with fourth order symmetrical profiles and a 47.18% increment in comparison with zero order profiles. By reducing the available solid volume to a lower value ($\overline{\sigma} = 0.15$), as it is evident in Fig. 6, the Nusselt number and the compared effectiveness decrease less for the symmetrical fin optimum geometries than for the asymmetrical ones, but the latter still perform the best.

In Fig. 6, the equivalent Nusselt number and compared effectiveness of the optimized geometries obtained for γ equal to 30 are also reported. It is evident that, when the ratio between the thermal conductivity of the solid and the fluid is a magnitude order lower, optimum asymmetrical fins perform noticeably worse than in the previous cases and just a little better than optimum symmetrical fins.

5. Conclusions

Some optimizations of the geometry of a finned dissipator cooled by laminar flow have been carried out for different situations by utilizing a finite element model in a genetic algorithm. In particular, for longitudinal fins with symmetrical and asymmetrical cross section, optimum lateral profiles have been found for given fin relative height and dissipator volume. The heat flux dissipated through the finned plate in a coolant

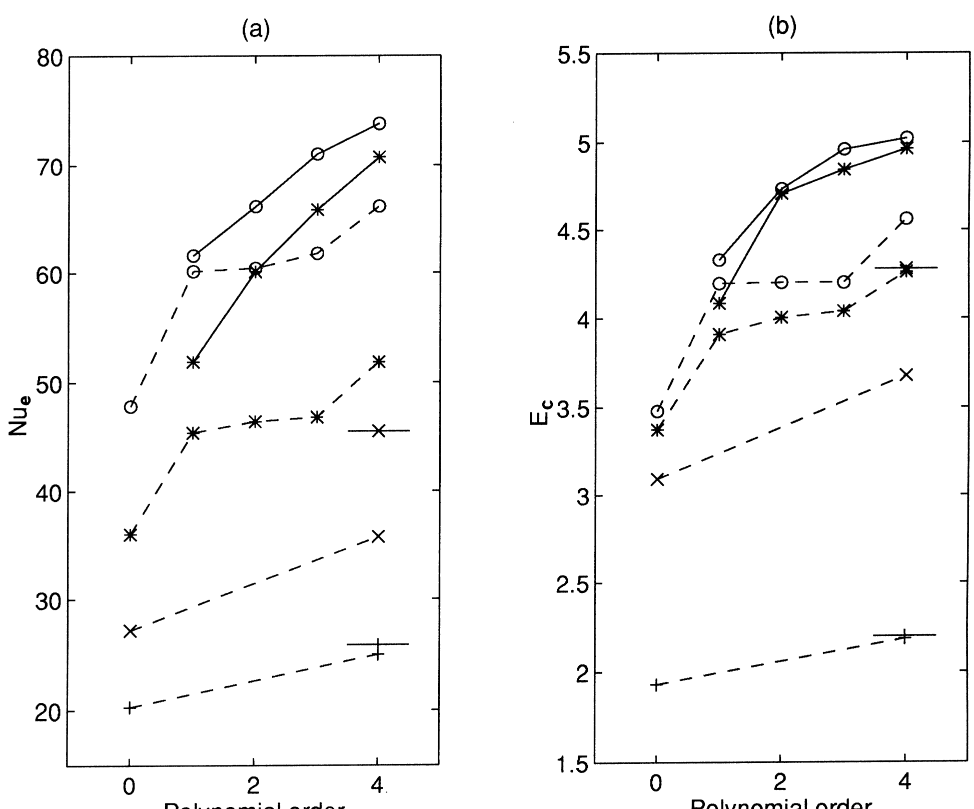

Fig. 6. Comparison between the performance of the optimized geometries with symmetrical (dashed line) and asymmetrical (continuous line) fins obtained by maximizing Nu_e (a) and E_c (b) for ϵ constrained to 0.5 α, γ equal to 300 (o, $\cdot$, $\times$) and 30 (+), $\overline{\sigma}$ unconstrained (o) and constrained to 0.30 ($\cdot$, +) and 0.15 ($\times$), and β unconstrained (o) and constrained to be no less than 0.05 ($\cdot$, $\times$, +).

fluid has been maximized with the lowest distance between the conduit wall or with the lowest hydraulic resistance.

Under particular conditions, noticeable improvements in the heat transfer have been observed for optimum fins with asymmetrical polynomial lateral profiles. Referring to the maximum global heat transfer coefficient which can be provided by zero order lateral profile fins, more than 95% increments have been obtained with fourth order asymmetrical fins. Moreover, performances being close to those of fourth and third order symmetrical profiles have been obtained with second and first order asymmetrical profiles, respectively.

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